4-4 Videos Guide

4-4a

- Mass of a planar lamina with shape D and density $\rho(x,y)$
  - $m = \iint_D \ \rho(x,y)\ dA$
- Total charge over an area with shape D and charge density $\sigma(x,y)$
  - $Q = \iint_D \ \sigma(x,y)\ dA$
- Moments
  - About the x-axis: $M_x = \iint_D \ y\ \rho(x,y)\ dA$
  - About the y-axis: $M_y = \iint_D \ x\ \rho(x,y)\ dA$
- Center of mass
  - $(\bar{x},\bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$
- Moments of inertia (second moments)
  - About the x-axis: $I_x = \iint_D \ y^2\ \rho(x,y)\ dA$
  - About the y-axis: $I_y = \iint_D \ x^2\ \rho(x,y)\ dA$
  - About the origin: $I_0 = \iint_D \ (x^2+y^2)\ \rho(x,y)\ dA = I_x + I_y$
- Radii of gyration
  - With respect to the x-axis: $\overline{\overline{y^2}} = \frac{I_x}{m}$
  - With respect to the y-axis: $\overline{\overline{x^2}} = \frac{I_y}{m}$
  - $(\bar{\bar{x}},\bar{\bar{y}})$ is the rotational analog to the center of mass

4-4b

- Probability
  - $P\big((X,Y)\ \epsilon\ D\big) = \iint_D \ \ f(x,y)\ dA$, where f is the joint density function of X and Y
  - Properties of a joint density function: $f(x,y) \geq 0$ and $\iint_{\mathbb{R}^2} f(x,y)\ dA = 1$
- Expected values for f a joint density function of random variables X and Y
  - x-mean: $\mu_1 = \iint_{\mathbb{R}^2} x\ f(x,y)\ dA$
  - y-mean: $\mu_2 = \iint_{\mathbb{R}^2} y\ f(x,y)\ dA$